

Planning models for optimal routing of radial distribution systems

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ABSTRACT

This paper presents three planning models for optimal routing of radial distribution systems. In the first two models, the cost function includes capital cost of lines, energy loss cost, and bays cost. The constraints equations include power balance equations, voltage drop equations, radiality equations, logic equations, thermal limit equations, and bus voltage limit equations. The first model considers the energy loss equation in its quadratic form while the second model approximates the energy loss equation of each cable size by a simple linear segment considering the economic loading of each cable size. In the third model, two sub-models are used where the first one gets the optimal radial network configuration regardless of the cable sizes and voltage constraints. In the second sub-model the best cable size on each selected line of the first model is determined to minimize the system costs while considering the bus voltage limit constraint and thermal limit constraint. Verification of the proposed planning models has been made using a real 11 kV 34-bus distribution network with 68 initial lines.

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NOMENCLATURE

n :	Number of load buses
m :	Number of feasible proposed lines
N_s :	Number of available cable sizes
N_f :	Number of outgoing lines from the substation
g :	Annual recovery rate of fixed cost
b :	Cost of a bay
l_i :	Length of line i
C^j :	Cost of cable size j per unit length
R^j :	Resistance of cable size j per unit length
I_i^j :	Current flow on line i and conductor size j
Z_i^j :	Zero-one integer variable associated with line i and conductor size j
F_t :	Total system cost per year
F_c :	Annualized cost of network feeders
F_b :	Annualized cost of bays
F_l :	Annualized cost of energy loss per year
l_s :	Loss factor
C_e :	Cost of unit energy in \$/kWh
S^j :	Energy loss cost of cable size j per unit current

$W_1(k)$:	Set of lines connected to bus k
D_k :	Demand at load bus k
x^j :	Reactance of cable size j per unit length
V_{i1} :	Voltage of bus i
\bar{k} :	Large positive number
$\cos\phi$:	System power factor

1. INTRODUCTION

The need for a reliable and economical electrical power system is increasing worldwide. The electrical distribution grid is a vital element in the electrical power system as it manages and delivers the generated electricity to customers [1, 2]. Distribution system planning (DSP) is a challenging task as it is a scheme that can achieve higher distribution reliability at a minimum total cost [2]. The DSP objective is to configure an optimum network that fulfills the load growth demand and satisfies the technical constraints at minimum total cost [3, 4]. The radial and the weakly meshed configurations are the most commonly used distribution system configurations [5, 6]. This paper is concerned with determining the optimal routing of radial distribution network configuration. The DSP optimal routing problem is about selecting and choosing feeders/paths/routes to reach an optimum configuration for the distribution radial network under study. An optimum configuration implies least capital cost and least energy losses cost of the distribution network.

The DSP optimization problem was solved using conventional mathematical techniques [2, 7-21] as well as heuristic techniques [22-27]. Though heuristic techniques give reasonable results in rapid computation time, still the mathematical techniques provide better optimal solutions. Furthermore, the DSP is a planning problem rather than a real time problem thus an accurate result obtained by conventional mathematical methods is more necessary than getting a faster solution [2]. Moreover, when conventional mathematical techniques fall in local optimum, it is clear that what was reached is local rather than global optimum solution. Mathematical optimization was implemented in [2] where the DSP was formulated and solved as a mixed integer linear programming problem (MILP). The cost function considered both the investment and outage costs to be minimized. The MILP method was compared to a meta-heuristic technique which is genetic algorithm (GA). The results revealed that MILP gave accurate optimal results whereas the GA provided a rapid reasonable solution. Moreover, the MILP implementation was simpler whereas the GA had difficulties in handling the constraints of the problem.

Moreover, Celli *et al.* [7] proposed a multi-criteria analysis to evaluate several options for a rural DSP. The multi-objective formulations of the problem considered the optimal sizing and siting of energy storage system. Furthermore, Wang *et al.* [8] proposed a multi-energy system expansion planning model with the aim to minimize the total cost of the system. A mixed integer second order cone programming model is deployed to optimize the size, the placement, and the type of all the infrastructure components of the multi-energy system. Besides, Liu *et al.* [9] proposed a mathematical model that took into consideration cost benefit analysis for the DSP problem. The proposed model considered the outage cost to quantify the reliability benefits based on consistence of economy. It was concluded that the cost benefit analysis consideration determined correctly the relation between economy and reliability thus yielding accurate calculations and results for the distribution grid planning. Additionally, Mohtasham *et al.* [10] solved the distribution expansion planning as a long term multi-year problem. The present value approach was used to solve this MINLP problem, co-optimizing the allocation of the DG in the network with the objective to minimize the total system cost. Also, Amjady *et al.* [11] presented a DSP model that uses non-linear convex AC power flow equations. The model considered the uncertainty in the load and the DG wind generation.

In this paper, three mathematical models are proposed to solve the DSP routing problem. The DSP routing problem is first formulated as a mixed integer quadratic programming (MIQP) problem, where an objective quadratic cost function that minimizes the capital cost and cost of energy losses is formed. Furthermore, a linearized model is introduced and the DSP is solved as a MILP problem. Additionally, a 2 sub-models system is introduced where the first sub-model gives an optimum configuration of the distribution network regardless of the cable sizes used for different branches in the configuration. Then, the second sub-model yields the optimum cable sizes associated with the network configuration obtained from the first sub-model solution. A real 11 kV 34-bus distribution system is used to test the validity of the 3 mathematical models proposed. Section 2 introduces the formulations of the MIQP mathematical model. Section 3 presents the MILP mathematical model formulations. Section 4 describes the two sub-models mathematical planning model. Section 5 describes how the proposed models are tested on a real 11 kV 34-node distribution system. The results, findings, and analysis of the studies are presented. Section 6 concludes the paper and envisions direction for future research.

2. MIXED INTEGER QUADRATIC PROGRAMMING (MIQP) MODEL

2.1. Cost function

The cost function to be minimized is the total annual cost which includes the fixed cost of lines, cost of bays, and the running cost of energy loss as follows:

$$F_t = F_c + F_b + F_l. \quad (1)$$

where:

$$F_c = g \sum_{i=1}^m l_i \sum_{j=1}^{N_s} C^j Z_i^j. \quad (2)$$

$$F_b = g \cdot b \cdot \sum_{i=1}^{N_f} \sum_{j=1}^{N_s} Z_i^j. \quad (3)$$

The losses cost is represented by a quadratic function in (4).

$$F_l = \bar{C} \sum_{i=1}^m l_i \sum_{j=1}^{N_s} R^j (I_i^j)^2. \quad (4)$$

where:

$$\bar{C} = 0.003 \times 8760 \cdot l_s \cdot C_e. \quad (5)$$

2.2. Constraints equations

The set of constraints equations required to be satisfied in order to get correct solutions are:

- a. Power balance equation for all load buses

For bus k:

$$\sum_{i \in W_1(k)} \sum_{j=1}^{N_s} I_i^j = D_i \quad (6)$$

- b. Voltage drop constraint for each line

This constraint relates the voltage drop on each line with the voltages of the two end buses of that line. For line k of the two end buses i_1 and i_2 where i_1 is the sending bus, this constraint is given as:

$$V_{i1} - V_{i2} - \sum_{j=1}^{N_s} l_k W^j I_k^j \leq \bar{k} (1 - \sum_{j=1}^{N_s} Z_k^j). \quad (7)$$

$$V_{i1} - V_{i2} - \sum_{j=1}^{N_s} l_k W^j I_k^j \geq \bar{k} (\sum_{j=1}^{N_s} Z_k^j - 1) \quad (8)$$

- c. Voltage limit constraint for all load buses

For bus i, it is given as:

$$V_i \geq V_{min} \quad (9)$$

where substation bus has a 1 p.u. voltage and V_{min} is minimum voltage permitted for load buses.

- d. Current limit constraint

For line k and cable size j, it is given as:

$$I_k^j \leq I_{max}^j Z_k^j, \text{ where } I_{max}^j \text{ is maximum current permitted on cable size } j.$$

- e. Radiality constraint

It is given as:

$$\sum_{i=1}^m \sum_{j=1}^{N_s} Z_i^j = n \quad (10)$$

- f. Logic constraint

It is given as:

$$\sum_{j=1}^{N_s} Z_k^j \leq 1 \quad (11)$$

The mathematical model (1-11), although highly accurate, suffers from the inherent difficulty found in mixed integer quadratic solution technique where it normally falls in local optimum even for moderate size networks. Also, the overlapping existing in the cost function of each adjacent cable sizes complicates the solution process. In general, if the above model fails to give global optimum for any problem, we can utilize the proposed model presented in the following section.

3. A MIXED INTEGER LINEAR PROGRAMMING (MILP) MODEL

This model makes a linearization of the non-linear cost function of the model described in section 2, in a simple and efficient manner as follows:

- Determine the economic loading range of each cable size as shown in Figure 1 where:
Economic loading range of size a is 0 to I'
Economic loading range of size b is I' to I''
Economic loading range of size c is I'' to I'''
- Get a one segment linear cost equation for the non-linear cost part of each cable size [28, 29] shown as blue solid segments in Figure 2. Now, the cost function for the new model will be as follows:

$$F_c = g \sum_{i=1}^m l_i \sum_{j=1}^{N_s} C^j Z_i^j \quad (12)$$

$$F_b = g \cdot b \cdot \sum_{i=1}^{N_f} \sum_{j=1}^{N_s} Z_i^j \quad (13)$$

$$F_l = \bar{C} \sum_{i=1}^m l_i \sum_{j=1}^{N_s} S^j I_i^j. \quad (14)$$

For line k, assuming three cable sizes, the equations for the new current limits of each cable size are:

$$I_k^a \leq I' Z_k^a \quad (15)$$

$$I_k^b \leq I'' Z_k^b \quad (16)$$

$$I_k^c \leq I''' Z_k^c \quad (17)$$

where I' , I'' , I''' are the new current limits for the three cable sizes as depicted in Figures 1 and 2.

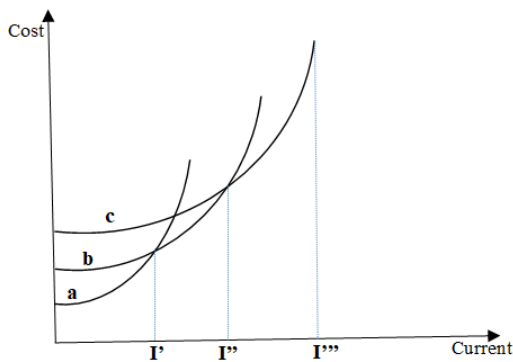


Figure 1. Costs and current limits for different cable sizes

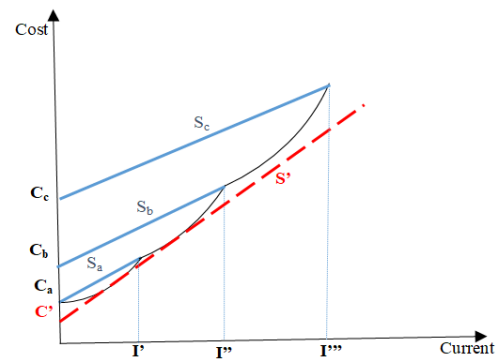


Figure 2. Linearization of fixed costs and energy loss costs for different cable sizes

4. TWO SUB-MODELS PLANNING MODEL

When the number of integer and linear variables of the above model is so large for real size distribution networks, the global optimum solutions are not reached. Thus, a new simple method has been developed to solve such large networks. The method divides the planning model into two parts as illustrated in Figure 3.

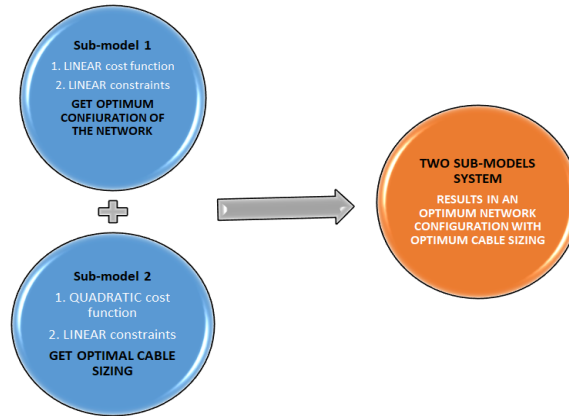


Figure 3. Two sub-models planning method framework

4.1. First part model

In this model, it is assumed that the cost function associated with all cable sizes of any line is approximated by one integer variable and one linear variable, this is depicted by the red dotted line shown in Figure 2. In this case, the cost function becomes:

$$F_t = g \cdot C' \cdot \sum_{i=1}^m l_i Z_i + g \cdot b \cdot \sum_{i=1}^{N_f} Z_i + \bar{C} \cdot S' \cdot \sum_{i=1}^m l_i I_i. \quad (18)$$

where C' is the fixed cost per km, and S' is the energy loss cost. In this case, the number of both integer and linear variables decrease largely such that the mixed integer linear programming technique can give a global solution within the above approximation.

The set of constraints are given simply as:

- a. Power balance equation for bus i

$$\sum_{j \in w_1(i)} I_j = D_i \quad (19)$$

- b. Current limit constraint for line i

$$I_i \leq I''' Z_i \quad (20)$$

where: I_i is the current flow on line i
 I''' is the current limit for the largest size cable

- c. Radiality constraint

This model gives the best radial configuration disregarding the voltage constraint and the type of cable to be used.

$$\sum_{i=1}^m Z_i = n. \quad (21)$$

4.2. Second part model

In this model, knowing the best radial configuration and the current flow on each selected line, it is required to get the best cable size of each line to minimize the cost function while satisfying the voltage constraint and the line limit constraint. The model is given as:

$$\text{Minimize } F_t = g \sum_{i=1}^m l_i \sum_{j=1}^{N_s} C^j Z_i^j + \bar{C} \sum_{i=1}^m I_i^2 l_i \sum_{j=1}^{N_s} R^j Z_i^j. \quad (22)$$

Subject to:

- a. Voltage constraint at each terminal point

$$\sum_{i=1}^{\bar{m}(t)} l_i I_i \sum_{j=1}^{N_s} W^j Z_i^j \leq \Delta V_{min}. \quad (23)$$

where $\bar{m}(t)$ is the set of lines found on the path from the substation to the terminal point t .

b. Current limit constraint for each line I of the planned radial network

$$I_i \leq \sum_{j=1}^{N_s} I_{max}^j Z_i^j. \quad (24)$$

c. Logic constraint for each line I of the radial planned network

$$\sum_{j=1}^{N_s} Z_i^j = 1. \quad (25)$$

5. RESULTS FOR THE PLANNING MODELS

5.1. Initial network

The verification of the proposed distribution system planning models is performed by the implementation to a real 11 kV 34-bus distribution system described and depicted in this section. The LINGO® 17.0 optimization software tool [30] is used to simulate the proposed model on an Intel® Core™ i7 @ 1.73 processor with a 4 GB installed memory on a 64-bit MS Windows® operating system. The initial network to be planned is shown in Figure 4.

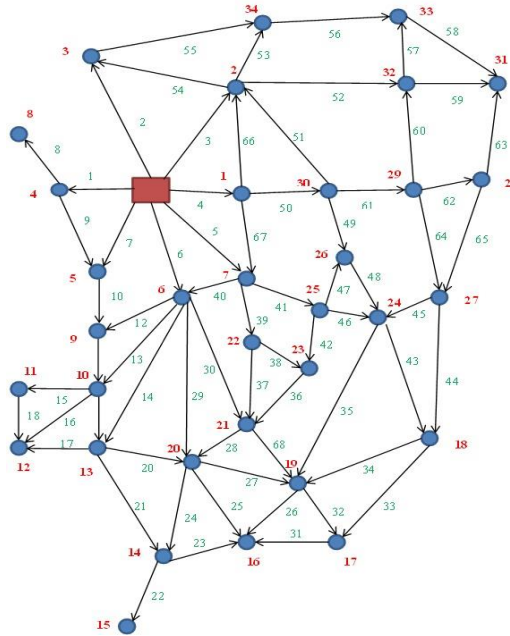


Figure 4. The initial 34-node distribution network

It is a part of the Qalyubia Governorate 11 kV distribution network; Qalyubia is an Egyptian Governorate situated north of Cairo. This distribution network to be planned contains 34 nodes and 68 feasible or proposed branches. The models formulation has been verified in the light of the following data:

- Cable sizes available are four. Their data is given in Table 1.
- $g=0.1$ and $b=10^6$ L.E.
- $C_e=0.5$ L.E./kwh and $l_s = 0.4$.
- Number of outgoing feeders from substation is 7.
- Load data and branch lengths are given in Table 2.
- Maximum voltage limit is 0.94 p.u. i.e. maximum voltage drop permitted for any terminal point is 0.06p.u.

Table 1. Cables impedance, current carrying capacity and cost

Size	I_t (Amp)	R (Ω /km)	X (Ω /km)	Cost (M.L.E./km)
a	150	1.1132	0.1371	0.159
b	260	0.4714	0.1415	0.358
c	315	0.2746	0.0968	0.48
d	422	0.1609	0.0968	0.635

Table 2. Load data and branches length

Line length (km)			Load power (p.u.)		
L ₁	1.1	L ₃₅	1.2	P ₁	0.024
L ₂	1	L ₃₆	0.9	P ₂	0.016
L ₃	0.9	L ₃₇	0.8	P ₃	0.016
L ₄	1.2	L ₃₈	0.8	P ₄	0.08
L ₅	1	L ₃₉	0.9	P ₅	0.019
L ₆	1.5	L ₄₀	1.1	P ₆	0.019
L ₇	0.9	L ₄₁	0.9	P ₇	0.022
L ₈	1.1	L ₄₂	0.8	P ₈	0.023
L ₉	0.8	L ₄₃	1	P ₉	0.025
L ₁₀	0.9	L ₄₄	1.2	P ₁₀	0.03
L ₁₁	0.8	L ₄₅	1.1	P ₁₁	0.025
L ₁₂	1	L ₄₆	0.9	P ₁₂	0.023
L ₁₃	1.2	L ₄₇	1	P ₁₃	0.03
L ₁₄	1.3	L ₄₈	1	P ₁₄	0.02
L ₁₅	1.2	L ₄₉	0.9	P ₁₅	0.024
L ₁₆	1.3	L ₅₀	1	P ₁₆	0.022
L ₁₇	0.9	L ₅₁	1.1	P ₁₇	0.024
L ₁₈	0.9	L ₅₂	1.5	P ₁₈	0.02
L ₁₉	0.8	L ₅₃	1.1	P ₁₉	0.022
L ₂₀	1.1	L ₅₄	1	P ₂₀	0.022
L ₂₁	1	L ₅₅	1.2	P ₂₁	0.024
L ₂₂	0.9	L ₅₆	1.2	P ₂₂	0.03
L ₂₃	0.9	L ₅₇	1	P ₂₃	0.025
L ₂₄	0.8	L ₅₈	0.9	P ₂₄	0.017
L ₂₅	1	L ₅₉	0.8	P ₂₅	0.025
L ₂₆	1.1	L ₆₀	1	P ₂₆	0.025
L ₂₇	1	L ₆₁	0.8	P ₂₇	0.022
L ₂₈	1.2	L ₆₂	1.1	P ₂₈	0.03
L ₂₉	0.9	L ₆₃	1.2	P ₂₉	0.02
L ₃₀	1.2	L ₆₄	1.3	P ₃₀	0.014
L ₃₁	0.9	L ₆₅	1.2	P ₃₁	0.026
L ₃₂	0.8	L ₆₆	1	P ₃₂	0.02
L ₃₃	1.1	L ₆₇	1	P ₃₃	0.028
L ₃₄	1	L ₆₈	0.9	P ₃₄	0.018

5.2. Mixed integer quadratic model application

The solution of the mixed integer quadratic model presented in section 2 has given the local optimum network shown in Figure 5. This network was solved yielding the costs: $F_c = 0.585$ ML.E., $F_b = 0.4$ ML.E., $F_l = 0.303$ ML.E., $F_i = 1.288$ ML.E.

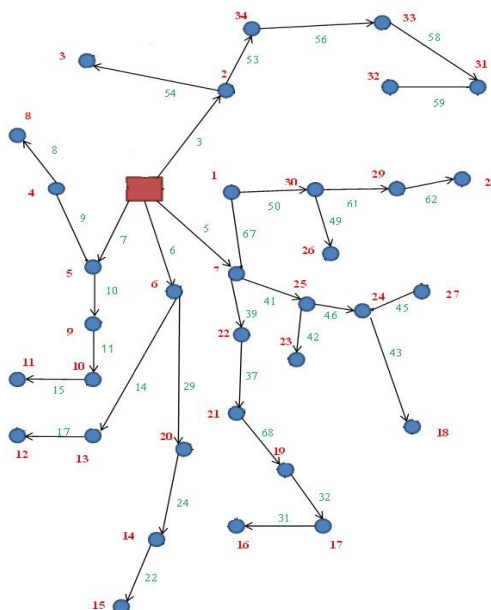


Figure 5. The solution of the 34-node distribution network using the mixed integer quadratic model

5.3. Mixed integer linear model application

The solution of the mixed integer linear model presented in section 3 has given the planned optimum network shown in Figure 6. The cost of the network are: $F_c = 0.63$ ML.E., $F_b = 0.4$ ML.E., $F_l = 0.0433$ ML.E., $F_t = 1.0733$ ML.E.

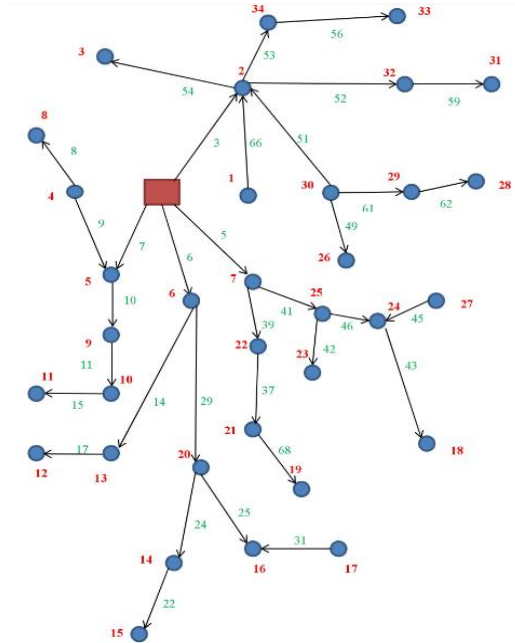


Figure 6. The solution of the 34-node distribution network using the mixed integer linear model

5.4. The two sub-model's method application

The solution of the network using the 2 sub-models method presented in section 4 yielded the optimum planned network shown in Figure 7. The costs of this network are: $F_c = 0.617$ ML.E., $F_b = 0.2$ ML.E., $F_l = 0.316$ ML.E., $F_t = 1.133$ ML.E.

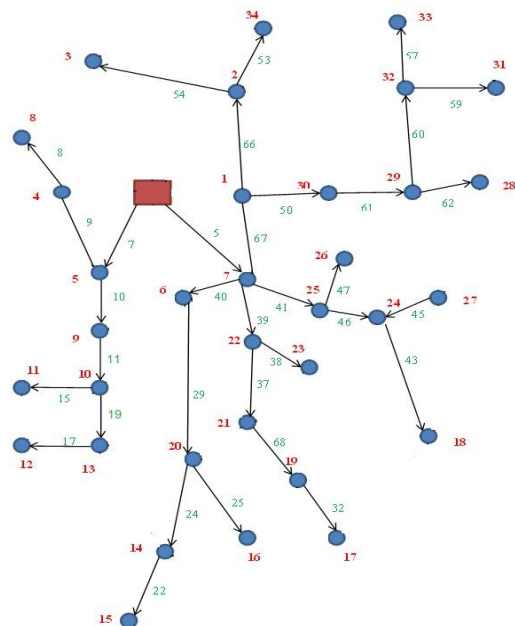


Figure 7. The solution of the 34-node distribution network using the two sub-models method

5.5. Results analysis

The analysis of the above outputs and findings reveals the following regarding the studied 34-node distribution network:

- The least total cost network (1.0733 MLE) is obtained when using the mixed integer linear model as illustrated in Figure 8.
- The MILP model and the 2 sub-models method give close results while the local optimum solution obtained by the MIQP method is relatively a higher cost solution as observed from Figure 8.
- It is observed from results depicted in Figures 5 and 7 that the MIQP and the 2 sub-models methods have selected the same 25 lines and differed only in 9 lines.



Figure 8. The costs obtained for the 34-node distribution radial network using the 3 proposed models

6. CONCLUSION

The paper has presented three methods for solving the optimal routing problem of the distribution system. The three models were tested using a real 11 kV 34-bus distribution network. The first method proposed is an accurate mixed integer quadratic programming model. When this method falls in a local optimum point, then the second method which is a mixed integer linear programming based method is used. This second mixed integer linear programming utilized the economic loading range of each cable size to approximate the quadratic energy loss cost equation to get a proper linear cost function for each cable size. When the second method fails to solve the large size networks, the third method can be used in which the planning problem is divided into two sub-models. The first sub-model searches for the optimum radial configuration and the second sub-model gets the best cable size on each route selected by the first sub-model. The implementation of the mixed integer linear model yielded an optimal planned radial distribution network with a least cost of $F_t = 1.0733$ MLE., whereas the highest cost network of $F_t = 1.288$ MLE. was obtained using the mixed integer quadratic model. The future work will be devoted to investigating the impact of allocating the shunt capacitors in radial distribution feeders and integrating these in the proposed mathematical models. The placement of capacitors is to be considered in planning to compensate for reactive power and to improve the voltage. Moreover, one of the challenges facing the distribution grid which is the integration of distributed energy resources is to be incorporated in future studies.

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